

Getting Started with A Level Maths and Further Maths



Maths Department

Bridging the gap from GCSE to A Level

The Calculator for A Level Maths

Exam boards expect students studying A level Maths to use a scientific calculator with an increased range of functions.

We recommend the *CASIO FX-991CW* for A Level Maths and Further Maths, and your lecturer will refer to this calculator during the course.

A more expensive graphical calculator (e.g. Casio CG100 or NumWorks) may be useful, but it's certainly not essential. If you are thinking of getting one, it may be worth waiting, as each year we order these through college and are able to get them much cheaper than their normal retail price.

Introduction

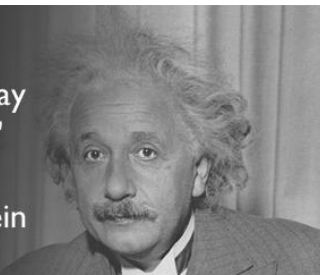
Congratulations on choosing to study A Level Maths. To help you prepare, this booklet will enable you to brush up on some of the skills you have learned at GCSE. You are going to need to use them from day 1. If you don't have a good grasp of the basics you need to work on them now so that you can start with confidence.

Do the questions in this booklet in pencil, then check your answers. If you get something wrong, revise the topic then try again. The aim is to get EVERYTHING right!

There are worked video solutions to every question to help you revise any difficult topics, just follow the links to youtube. Or search for 'Truro College getting started with AS Maths'.

"It's not that I'm so smart, it's just that I stay with problems longer."

—Albert Einstein



Studying A Level Maths is about learning how to solve problems, and getting stuck is part of the learning process. You should expect to get stuck while working through this booklet, but these are all GCSE techniques that you will need to master.

There are loads of great resources on the internet to help you, but if you get stuck we recommend **ExamSolutions.net** which contains video tutorials for all GCSE Higher content. We also recommend using this site throughout the A Level course.

<http://www.examsolutions.net/gcse-maths/>



Tip - Download a QR code reader app if you have a smartphone or tablet. The QR codes throughout this booklet link to helpful websites and tutorials, but if you don't have a smartphone or tablet with this function, you should be able to find them by name on the relevant websites.

There will be a TEST in your first week at college based on the material in this booklet. Please bring the completed booklet with you to your first maths lesson.

If you want a more comprehensive revision guide we recommend:

- CGP – Year 1 Complete Revision & Practice

There are video solutions to all the questions in this booklet:



<https://www.youtube.com/playlist?list=PLW83lrLuu01bYQtdnIHeRnHD4RRDoMrX6>

Please use it to help with any questions you get stuck on.

GCSE Maths

If you're naturally good at maths you can do well without much extra studying

It's the answer that matters most, but you should show working

You have an exercise book to keep all your work together

Nobody minds how you set out your workings so long as you get there in the end

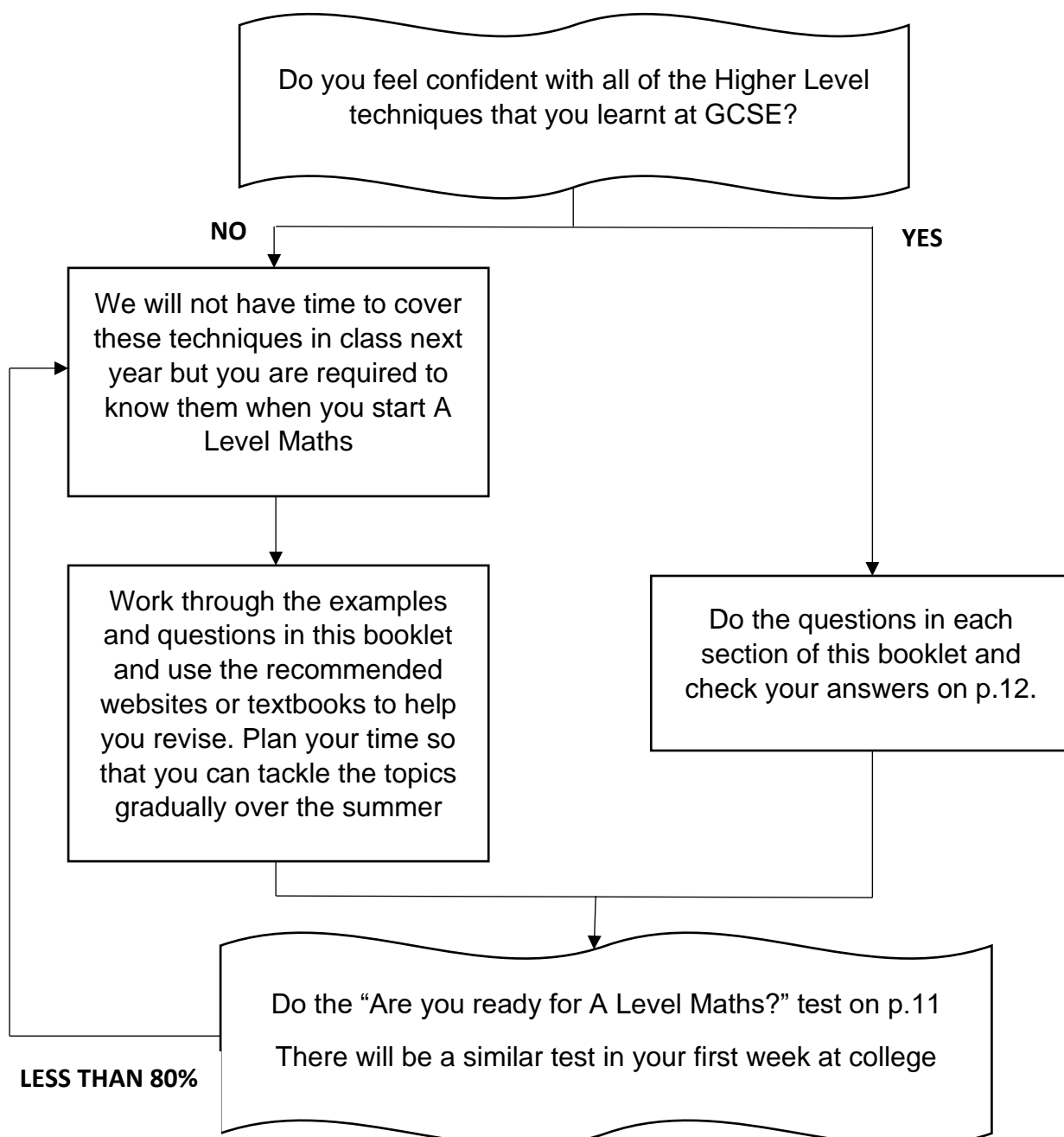
A Level Maths

You will do a lot of study outside of class

It's the method that matters, not the answer. Often you are given the answer but need to show steps in the method.

You will need to buy a folder and some dividers to organise your notes. It is a good idea to keep your 'neat' notes separate from your rough work so you can find them easily for revision.

How you present your work can make a big difference to whether you get the right answer at all and whether anyone can understand your method.



1. Fractions

You need to be really confident with numerical fractions so that you know what to do with algebraic ones.

Multiplication: $\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$ and $2 \times \frac{3}{5} = \frac{2}{1} \times \frac{3}{5} = \frac{6}{5}$ NOT $\frac{6}{10}$!!!

When using algebra you will make fewer mistakes if you write things next to each other, like $3x$ rather than $3 \times x$, and use brackets.

So, using algebra:

$$2x \left(\frac{3x}{4} \right) = \left(\frac{2x}{1} \right) \left(\frac{3x}{4} \right) = \frac{6x^2}{4} = \frac{3x^2}{2}$$

(Always simplify fractions by dividing top and bottom by any common factors as far as possible)

Division: $\frac{8}{3} \div \frac{2}{3} = \frac{8}{3} \times \frac{3}{2} = \frac{8 \times 3}{3 \times 2} = \frac{8}{2} = 4$ and $5x \div \frac{1}{x} = \left(\frac{5x}{1} \right) \left(\frac{x}{1} \right) = 5x^2$

Addition and subtraction - start by making the denominators the same:

$$\frac{5}{4} + \frac{3}{2} = \frac{5}{4} + \frac{6}{4} = \frac{11}{4} \quad \left(\text{At A Level we prefer this as an 'improper' fraction NOT } 2\frac{3}{4} \right)$$

Where there is no obvious common denominator you can make one by multiplying the denominators together. Whatever you do to the bottom of a fraction, you have to do the same to the top.

e.g. $\frac{2x}{5} - \frac{1}{2} = \frac{2x \times 2}{5 \times 2} - \frac{1 \times 5}{2 \times 5} = \frac{4x}{10} - \frac{5}{10} = \frac{4x-5}{10}$

Exercise 1

Without a calculator, work these out as a single simplified fraction and check your answers at the end of the booklet on p.12 (tick the box when you have got it right).

1. $\frac{3}{4} \times \frac{2}{5}$ <input type="checkbox"/>	2. $2 + \frac{3}{5}$ <input type="checkbox"/>	3. $\frac{3}{2} \div \frac{1}{4}$ <input type="checkbox"/>
4. $\frac{2/7}{4}$ <input type="checkbox"/>	5. $\frac{3x}{5} \times 4$ <input type="checkbox"/>	6. $\frac{1}{x} + \frac{2}{x}$ <input type="checkbox"/>
7. $\frac{5}{3/2}$ <input type="checkbox"/>	8. $\frac{2/3}{3/4}$ <input type="checkbox"/>	9. $\left(\frac{3}{8} \div \frac{1}{4} \right) \times \left(\frac{2x}{3} \right)$ <input type="checkbox"/>
10. $\frac{3}{x} + \frac{2}{x^2}$ (Hint: multiply the top and bottom of $\frac{3}{x}$ by x first) <input type="checkbox"/>	11. $\left(\frac{3}{2} \times \frac{1}{4} \right) + 3$ <input type="checkbox"/>	12. $\frac{2x+7}{2} - \frac{3}{5}$ <input type="checkbox"/>

Video Solutions:

<https://www.youtube.com/watch?v=1Gh7BWlZ3o8&list=PLW83lrLuu01bYQtdnHeRnHD4RRDoMrX6&index=2>



2. Indices

You will literally be using indices all the time at AS Level so get to grips with them now!

These are the **rules of indices** you need to know:

$$a^m a^n = a^{m+n} \quad \frac{a^m}{a^n} = a^{m-n} = a^n \div a^m \quad (a^m)^n = a^{mn} = (a^n)^m$$

$$(ab)^n = a^n b^n \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad a^0 = 1 \quad a^1 = a \quad a^{-1} = \frac{1}{a}$$

A **negative** power means a **reciprocal** e.g. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ and $\left(\frac{1}{2}\right)^{-2} = \left(\frac{2}{1}\right)^2 = 4$

A **fractional** power indicates a **root** e.g. $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$ (since $2 \times 2 \times 2 = 8$)

Example 1:

cube square root

$$16^{\frac{3}{2}} = (\sqrt{16})^3 = 4^3 = 64$$

Example 2:

reciprocal to the power of 4

cube root

$$8^{-\frac{4}{3}} = \left(\frac{1}{8}\right)^{\frac{4}{3}} = \left(\frac{1}{\sqrt[3]{8}}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

(Hint: – do the reciprocal first, then the root, then the top power)

Example 3 (harder example): $(9x^4y^3)^{\frac{1}{2}} = 3x^2y^{\frac{3}{2}}$

(Hint: - Everything in the bracket needs to be square rooted, the 9 the x^4 and the y^3 .)

In order to do this, use the index rule: $(a^m)^n = a^{mn}$.

Exercise 2a – Do this exercise without a calculator

Simplify the following – leave your answer in the form: a^n

1. $b^4 \times b^3$	2. $a^5 \div a^3$	3. $(x^3)^2$
<input type="text"/>	<input type="text"/>	<input type="text"/>

Evaluate the following **without using a calculator** (i.e. find the value of)

4. $\left(\frac{2}{5}\right)^3$	5. $27^{\frac{1}{3}}$	6. $9^{\frac{3}{2}}$
<input type="text"/>	<input type="text"/>	<input type="text"/>
7. $81^{-\frac{1}{4}}$	8. $\left(\frac{2}{3}\right)^{-2}$	9. $\sqrt{\frac{4}{9}}$
<input type="text"/>	<input type="text"/>	<input type="text"/>

Video Solutions:

<https://www.youtube.com/watch?v=saVs9P2Gi80&list=PLW83lrLuu01bYQtdnlHeRnHD4RRDoMrX6&index=3>



Indices – Expressing terms in the form ax^n

It is often necessary to write expressions in the form (number) x^{power} or ax^n

Common mistake:

$$\frac{1}{3x^2} = 3x^{-2} \quad \text{WRONG!}$$

Actually: $\frac{1}{3x^2} = \left(\frac{1}{3}\right)\left(\frac{1}{x^2}\right) = \frac{1}{3}x^{-2}$

Common mistake:

$$\sqrt{4x} = 4x^{\frac{1}{2}} \quad \text{WRONG!}$$

Actually: $\sqrt{4x} = \sqrt{4}\sqrt{x} = 2x^{\frac{1}{2}}$

One important technique is 'sliding' the number away from the x term so that you can simplify them separately.

Example 1: $\frac{2}{x} = 2 \times \frac{1}{x} = 2x^{-1}$

Example 2: $\frac{6}{5x^2} = \left(\frac{6}{5}\right)\left(\frac{1}{x^2}\right) = \frac{6}{5}x^{-2}$

You can split the numerator of a fraction to make two separate terms, but you can never do this with the denominator

Example 3: $\frac{2+x}{\sqrt{x}} = \frac{2}{\sqrt{x}} + \frac{x}{\sqrt{x}} = 2\left(\frac{1}{\sqrt{x}}\right) + \frac{x^1}{x^{\frac{1}{2}}} = 2x^{-\frac{1}{2}} + x^{\frac{1}{2}}$

BUT $\frac{x^2}{x+1} \neq \frac{x^2}{x} + \frac{x^2}{1}$ THIS IS **WRONG!** In fact this fraction cannot be simplified.

Watch this video on
[examsolutions.net](https://www.examsolutions.net/Indices-Expressing-in-the-form-axn)
Indices – Expressing
in the form ax^n



Exercise 2b – write these expressions in the form ax^n

1. $5\sqrt{x}$ <input type="text"/>	2. $\frac{2}{x^3}$ <input type="text"/>	3. $\frac{3}{\sqrt{x}}$ <input type="text"/>
4. $\frac{\sqrt{x}}{5}$ <input type="text"/>	5. $\left(\frac{2x}{3}\right)^2$ <input type="text"/>	6. $\frac{1}{\sqrt[3]{x}}$ <input type="text"/>
7. $(2\sqrt{x})^3$ <input type="text"/>	8. $\frac{4}{3x^5}$ <input type="text"/>	9. $\frac{\sqrt{x}}{3x}$ <input type="text"/>
10. $\frac{3x^2}{\sqrt{x}}$ <input type="text"/>	11. $\frac{x-2}{x^2}$ (The answer to this one has two separate terms) <input type="text"/>	

Simplify the following - leave your answer in the form: Ax^ny^m (where a, n, m are constants).

12. $\frac{1}{2x^2y}$ <input type="text"/>	13. $(27x^6y^5)^{\frac{1}{3}}$ <input type="text"/>	14. $\left(\frac{16x^2}{y}\right)^{-\frac{1}{4}}$ <input type="text"/>
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Video Solutions:

<https://www.youtube.com/watch?v=hTzkdeLeaD0&list=PLW83lrLuu01bYQtdnlHeRnHD4RRDoMrX6&index=4>



3. Surds

A surd is an irrational root e.g. $\sqrt{2}$, $\sqrt{3}$ but not $\sqrt{9}$ because $\sqrt{9} = 3$.

How to simplify surds:

$$\sqrt{ab} = \sqrt{a}\sqrt{b} \quad \text{e.g. } \sqrt{20} = \sqrt{4 \times 5} = \sqrt{4}\sqrt{5} = 2\sqrt{5}$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \quad \text{e.g. } \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$$

Look for multiples of square numbers
e.g. 4, 9, 16 etc

[BBC Bitesize – Simplifying Surds](#)



$$\begin{aligned} \text{Example: } \sqrt{75} + 2\sqrt{12} &= \sqrt{25 \times 3} + 2\sqrt{4 \times 3} = \sqrt{25}\sqrt{3} + 2\sqrt{4}\sqrt{3} \\ &= 5\sqrt{3} + 4\sqrt{3} = 9\sqrt{3} \end{aligned}$$

Rationalising the denominator:

This means re-writing a fraction so that there is no surd on the bottom. We do this by multiplying both top and bottom by the surd on the bottom.

$$\text{Example 1: } \frac{1}{\sqrt{5}} = \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{\sqrt{5}\sqrt{5}} = \frac{\sqrt{5}}{5}$$

Remember: $\sqrt{5}\sqrt{5} = (\sqrt{5})^2 = 5$ NOT 25!

Where the denominator has two parts we multiply the top and bottom by the whole denominator but we need to change the sign in the middle.

$$\begin{aligned} \text{Example 2: } \frac{3}{1+\sqrt{2}} &= \frac{3}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{3(1-\sqrt{2})}{(1+\sqrt{2})(1-\sqrt{2})} = \frac{3-3\sqrt{2}}{1-\sqrt{2}+\sqrt{2}-\sqrt{2}\sqrt{2}} \\ &= \frac{3-3\sqrt{2}}{1-2} = \frac{3-3\sqrt{2}}{-1} = -3 + 3\sqrt{2} \end{aligned}$$

Note: dividing by -1 has the same effect as multiplying by -1
i.e. it changes all the signs

Example 3: Write $\frac{1}{3-\sqrt{3}}$ in the form $a + b\sqrt{3}$

$$\frac{1}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{3+\sqrt{3}}{(3-\sqrt{3})(3+\sqrt{3})} = \frac{3+\sqrt{3}}{9-3\sqrt{3}+3\sqrt{3}-\sqrt{3}\sqrt{3}} = \frac{3+\sqrt{3}}{9-3} = \frac{3+\sqrt{3}}{6} = \frac{3}{6} + \frac{\sqrt{3}}{6} = \frac{1}{2} + \frac{1}{6}\sqrt{3}$$

Exercise 3 -

Write in the form $a\sqrt{b}$

Rationalise the denominator

1. $\sqrt{27}$ <input type="text"/>	4. $\sqrt{20} - 3\sqrt{45}$ <input type="text"/>	6. $\frac{2}{\sqrt{3}}$ <input type="text"/>
2. $\sqrt{48}$ <input type="text"/>	5. $\sqrt{200} + \sqrt{18} - 2\sqrt{50}$ <input type="text"/>	7. $\frac{1}{1+\sqrt{2}}$ <input type="text"/>
3. $\frac{\sqrt{12}}{2}$ <input type="text"/>		8. $\frac{3}{4-\sqrt{2}}$ <input type="text"/>

Video Solutions:

<https://www.youtube.com/watch?v=Ugrln-qDyfM&list=PLW83lrLuu01bYQtdnIHeRnHD4RRDoMrX6&index=5>



4. Quadratics

Quadratics are everywhere in A Level Maths! However, you should already be pretty good at the basic techniques so just keep practising.

Factorisation

These two numbers multiply to give + 6...

$$x^2 - 5x + 6 = (x - 3)(x - 2) \quad \dots \text{and add together to give } -5$$

Remember – not all quadratics can be factorised!

Difference of two squares (special kind of factorisation)

$$a^2 - b^2 = (a + b)(a - b)$$

Example 1: $9 - x^2 = (3 + x)(3 - x)$

Example 2: $4x^2 - 25 = (2x + 5)(2x - 5)$

Removing a common factor

Example 3: $10x^2 - 5x = 5x(2x - 1)$

Both terms have a common factor of 5x, so we can factorise with a single bracket.

The solutions would then be $x = 0$ and $x = \frac{1}{2}$ (set each term equal to 0 and solve)

So $5x = 0$, and $2x - 1 = 0$, giving x solutions of 0 and $\frac{1}{2}$

The quadratic formula

You need to learn this formula as you are not given it in A Level exams.

If $ax^2 + bx + c = 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (Note - Your answer might involve surds)

Exercise 4a

Factorise the following quadratics. The first one has been partially completed for you. Check your answers by expanding the brackets and seeing if you end up with the original quadratic.

1. $x^2 + 2x - 15$ $= (x - 3)(\quad)$ <div></div>	2. $x^2 - 9x - 10$ <div></div>	3. $6x^2 + 2x$ (hint – take out the common factors) <div></div>
4. $49 - 4x^2$ <div></div>	5. $2x^2 + 5x - 3$ <div></div>	6. $4x^2 + 4x + 1$ <div></div>

Solve using the quadratic formula without a calculator (where necessary leave in surd form):

7. $x^2 - 5x + 4 = 0$ <div></div>	8. $3x^2 + 2x - 1 = 0$ <div></div>	9. $x^2 = 3x + 2$ (hint: rearrange to get = 0 first) <div></div>
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Factorise & solve

10. $10x^2 - 2x = 0$ <div></div>	11. $9x - 27x^2 = 0$ <div></div>	12. $14x^2 - 21x = 0$ <div></div>
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Video Solutions:

https://www.youtube.com/watch?v=wbW7Z_wrngA&list=PLW83lrLuu01bYQtdnlHeRnHD4RRDoMrX6&index=6

There are other techniques for factorising quadratics if you can't do it 'by inspection' (i.e. just looking at the numbers). [Watch this video from khanacademy.org to learn a different approach called 'grouping'](https://www.khanacademy.org/learn-a-different-approach-called-grouping)

Always check the sign in front of a, b, and c, not just the number



Completing the square

Some quadratics are 'perfect squares' e.g. $x^2 + 4x + 4 = (x + 2)(x + 2) = (x + 2)^2$

Most quadratics are not like this, but can be written as a square that is 'adjusted' slightly.

e.g. $x^2 + 4x + 7$ the first two terms are the same as above so try $(x + 2)^2$, but this gives + 4 as the constant and we want + 7, hence $x^2 + 4x + 7 = (x + 2)^2 - 4 + 7 = (x + 2)^2 + 3$

In general:

$$x^2 + bx + c = (x + \text{half of } b)^2 - (\text{half of } b)^2 + c$$

Example 1: $x^2 + 6x + 2 = (x + 3)^2 - (3)^2 + 2 = (x + 3)^2 - 7$

Example 2: $x^2 - 4x + 3 = (x - 2)^2 - (-2)^2 + 3 = (x - 2)^2 - 4 + 3 = (x - 2)^2 - 1$

Example 3: $x^2 + 5x - 2 = \left(x + \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 - 2 = \left(x + \frac{5}{2}\right)^2 - \frac{25}{4} - 2 = \left(x + \frac{5}{2}\right)^2 - \frac{33}{4}$

Exercise 4b

By completing the square, write these quadratic expressions in the form $(x + p)^2 + q$

[examsolutions.net](https://www.examsolutions.net/Completing-the-Square)
– Completing the
Square



1. $x^2 + 8x + 7$ <input type="text"/>	2. $x^2 - 2x - 15$ <input type="text"/>	3. $x^2 + 6x + 10$ <input type="text"/>
4. $x^2 + 12x + 100$ <input type="text"/>	5. $x^2 - 3x - 1$ <input type="text"/>	6. $x^2 - \frac{1}{2}x + 1$ <input type="text"/>

Video Solutions:

<https://www.youtube.com/watch?v=QbtqWENavr0&list=PLW83lrLuu01bYQtdnlHeRnHD4RRDoMrX6&index=7>

Going a step further:

You can solve a quadratic equation in this way:

e.g. $x^2 - 4x - 5 = 0$

1. Complete the square:

$$(x - 2)^2 - 9 = 0$$

2. Put the number of the right hand side

$$(x - 2)^2 = 9$$

3. Square root both sides (remembering the \pm sign!)

$$x - 2 = \pm 3$$

4. Add 2 to both sides to get TWO answers

$$x = 2 \pm 3 \text{ so } x = 5 \text{ or } x = -1$$



Exercise 4c Solve by completing the square

1. $x^2 + 6x - 7 = 0$ <input type="text"/>	2. $x^2 - 2x - 3 = 0$ <input type="text"/>	3. $x^2 + 5x = -6$ <input type="text"/>
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Video Solutions:

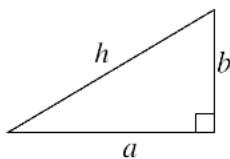
<https://www.youtube.com/watch?v=37eoFAIq2IQ&list=PLW83lrLuu01bYQtdnlHeRnHD4RRDoMrX6&index=8>



5. Trigonometry

This won't pop up until a bit later in the year, but it is stuff you should already know!

formulae
for
right-
angled
triangles



Pythagoras's Theorem

$$a^2 + b^2 = h^2$$

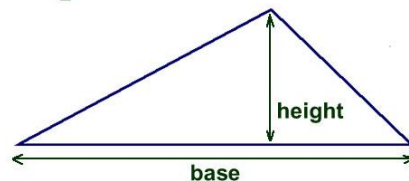
Trigonometric Ratios

$$\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$$

$$\cos(\theta) = \frac{\text{adj}}{\text{hyp}}$$

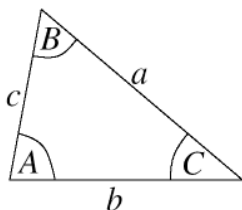
$$\tan(\theta) = \frac{\text{opp}}{\text{adj}}$$

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{perpendicular height}$$



You are expected to learn the sine rule and cosine rule. These will not be given to you on the exam paper

formulae
for all
triangles



Sine Rule

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \quad \text{or} \quad \frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

(for finding sides)

(for finding angles)

Cosine Rule

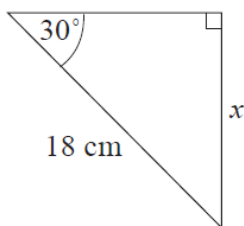
$$a^2 = b^2 + c^2 - 2bc \cos A \quad \text{or} \quad \cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

(for finding sides)

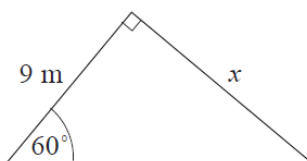
(for finding angles)

Exercise 5 – find the side marked x or the angle marked θ in each triangle to 1 d.p.

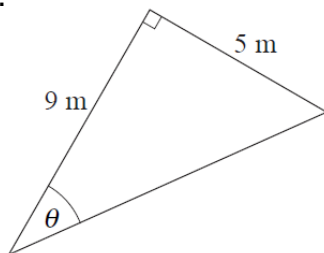
1.



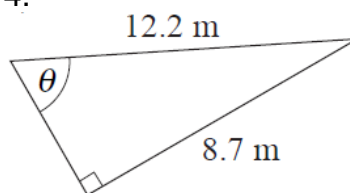
2.



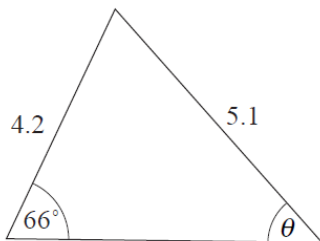
3.



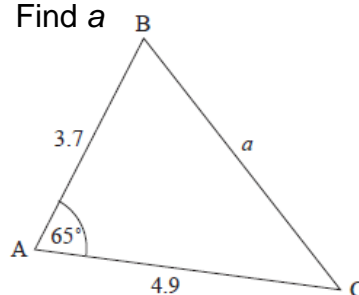
4.



5.



6. Find a



Video Solutions:

https://www.youtube.com/watch?v=qpgP_PuZToQ&list=PLW83lrLuu01bYQtdnlHeRnHD4RRDoMrX6&index=9



ARE YOU READY FOR A LEVEL MATHS? - PRACTICE TEST

When you start the course we will give you a test on the material in this booklet to check that you are ready to start A Level. Try this similar test in exam conditions (write on lined paper, not this booklet) then mark it using the answers at the back of the booklet and give yourself a score. You should aim for over 80% but certainly nothing less than 60% should be a worry. Go back to the exercises containing the questions you got wrong, re-watch the video solutions then try this test again in a few days' time. Repeat this process until you feel confident to start the A Level course.

Time: 1 hour. No calculator allowed except for Q9.

1. Write as a single fraction:

a) $\frac{3}{2/5}$

b) $\left(\frac{3x}{2} \div \frac{5}{3}\right) \times \frac{1}{3}$

2. Evaluate:

a) $16^{-\frac{1}{4}}$

Simplify fully:

b) $\left(\frac{16}{x^3}\right)^{-\frac{1}{2}}$

Quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Cosine rule:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

3. Write in the form ax^n :

a) $\frac{2}{3x}$

b) $\frac{4\sqrt{x}}{5}$

4. Simplify:

a) $\sqrt{32}$

b) $\sqrt{20} + 2\sqrt{45} - 3\sqrt{80}$

5. Rationalise the denominator:

a) $\frac{1}{\sqrt{2}}$

b) $\frac{5}{2-\sqrt{3}}$

6. Solve these quadratics by factorising them:

a) $x^2 - 5x - 24 = 0$

b) $9x^2 - 4 = 0$

7. Solve using the quadratic formula (leave your answer in surd form if necessary):

a) $6x^2 + x - 1 = 0$

b) $x^2 - 7x + 9 = 0$

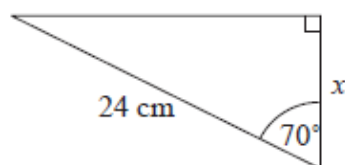
8. Write in the form $(x + p)^2 + q$ (i.e. complete the square):

a) $x^2 + 2x - 6$

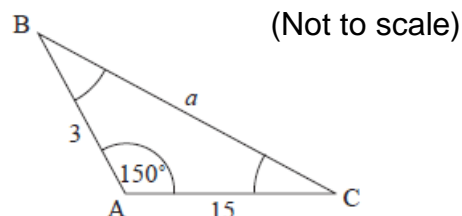
b) $x^2 + 3x + \frac{1}{4}$

9. Find the side marked x or a to 1 d.p.:

a)



b)



10. a) Simplify $3^3 \times 3^n$

b) Hence write $27 \times (3^{n+1})$ in the form 3^p

Score

/40

Answers to Exercises

Exercise 1 - Fractions

- | | | | | | |
|-------------------|-------------------|--------|------------------------|--------------------|-------------------------|
| 1. $\frac{3}{10}$ | 2. $\frac{13}{5}$ | 3. 6 | 4. $\frac{1}{14}$ | 5. $\frac{12x}{5}$ | 6. $\frac{3}{x}$ |
| 7. $\frac{10}{3}$ | 8. $\frac{8}{9}$ | 9. x | 10. $\frac{3x+2}{x^2}$ | 11. $\frac{27}{8}$ | 12. $\frac{10x+29}{10}$ |

Exercise 2a - Indices

- | | | | | | | |
|------------------|------------------|----------|--------------------|------|-------|------------------|
| 1. b^7 | 2. a^2 | 3. x^6 | 4. $\frac{8}{125}$ | 5. 3 | 6. 27 | 7. $\frac{1}{3}$ |
| 8. $\frac{9}{4}$ | 9. $\frac{2}{3}$ | | | | | |

Exercise 2b – Writing as ax^n

- | | | | | | |
|---------------------------|--------------------------------------------------|----------------------------------|---------------------------------|------------------------|-------------------------------|
| 1. $5x^{\frac{1}{2}}$ | 2. $2x^{-3}$ | 3. $3x^{-\frac{1}{2}}$ | 4. $\frac{1}{5}x^{\frac{1}{2}}$ | 5. $\frac{4}{9}x^2$ | 6. $x^{-\frac{1}{3}}$ |
| 7. $8x^{\frac{3}{2}}$ | 8. $\frac{4}{3}x^{-5}$ | 9. $\frac{1}{3}x^{-\frac{1}{2}}$ | 10. $3x^{\frac{3}{2}}$ | 11. $x^{-1} - 2x^{-2}$ | 12. $\frac{1}{2}x^{-2}y^{-1}$ |
| 13. $3x^2y^{\frac{5}{3}}$ | 14. $\frac{1}{2}x^{-\frac{1}{2}}y^{\frac{1}{4}}$ | | | | |

Exercise 3 - Surds

- | | | | | |
|--------------------------|--------------------|------------------------------|-----------------|----------------|
| 1. $3\sqrt{3}$ | 2. $4\sqrt{3}$ | 3. $\sqrt{3}$ | 4. $-7\sqrt{5}$ | 5. $3\sqrt{2}$ |
| 6. $\frac{2\sqrt{3}}{3}$ | 7. $-1 + \sqrt{2}$ | 8. $\frac{12+3\sqrt{2}}{14}$ | | |

Exercise 4a – Factorising and the quadratic formula

- | | | |
|-----------------------------------------------|--------------------------------------|-----------------------------------------------------------|
| 1. $(x-3)(x+5)$ | 2. $(x-10)(x+1)$ | 3. $2x(3x+1)$ |
| 4. $(7+2x)(7-2x)$ (Difference of two squares) | | |
| 5. $(2x-1)(x+3)$ | 6. $(2x+1)^2$ | |
| 7. $x=4$ or $x=1$ | 8. $x=\frac{1}{3}$ or $x=-1$ | 9. $x=\frac{3+\sqrt{17}}{2}$ or $x=\frac{3-\sqrt{17}}{2}$ |
| 10. $2x(5x-1)$, so $x=0$ or $x=\frac{1}{5}$ | 11. $9x(1-3x)$, so $x=0$ or $x=1/3$ | |
| 12. $7x(2x-3)$, so $x=0$ or $x=\frac{3}{2}$ | | |

Exercise 4b – Completing the square

- | | | |
|-------------------|--------------------------------------------------|---------------------------------------------------|
| 1. $(x+4)^2 - 9$ | 2. $(x-1)^2 - 16$ | 3. $(x+3)^2 + 1$ |
| 4. $(x+6)^2 + 64$ | 5. $\left(x-\frac{3}{2}\right)^2 - \frac{13}{4}$ | 6. $\left(x-\frac{1}{4}\right)^2 + \frac{15}{16}$ |

Exercise 4c – Solving by completing the square

- | | | |
|--------------------|--------------------|---------------------|
| 1. $x=-7$ or $x=1$ | 2. $x=3$ or $x=-1$ | 3. $x=-2$ or $x=-3$ |
|--------------------|--------------------|---------------------|

Exercise 5 - Trigonometry

- | | | |
|------------------------|------------------------------------|--------------------------|
| 1. $x=9$ | 2. $x=15.6$ | 3. $\theta s=29.1^\circ$ |
| 4. $\theta=45.5^\circ$ | 5. $\theta=48.8^\circ$ (Sine Rule) | 6. $a=4.7$ (Cosine Rule) |

ARE YOU READY FOR A LEVEL MATHS? - SOLUTIONS

For each part, give yourself 2 marks for a perfect answer (including working), 1 mark if you used the correct method but made a mistake and 0 marks for doing it totally wrong! The total test is out of 40 and **anything below 24/40 is worrying and means you must go back to the exercises and try again to master the techniques, using the tips on page 2 of the booklet for help.**

$$1. a) \frac{3}{2/5} = \frac{3/1}{2/5} = \frac{3}{1} \times \frac{5}{2} = \frac{15}{2} \quad b) \left(\frac{3x}{2} \div \frac{5}{3}\right) \times \left(\frac{1}{3}\right) = \left(\frac{9x}{10}\right) \times \left(\frac{1}{3}\right) = \frac{3x}{10}$$

$$2. a) (16)^{-\frac{1}{4}} = \frac{1}{2} \quad b) \left(\frac{16}{x^3}\right)^{-\frac{1}{2}} = \left(\frac{x^3}{16}\right)^{\frac{1}{2}} = \frac{x^{\frac{3}{2}}}{4}$$

$$3. a) \left(\frac{2}{3}\right)\left(\frac{1}{x}\right) = \frac{2}{3}x^{-1} \quad b) \frac{4}{5}x^{\frac{1}{2}}$$

$$4. a) \sqrt{16}\sqrt{2} = 4\sqrt{2}$$

$$b) \sqrt{4}\sqrt{5} + 2\sqrt{9}\sqrt{5} - 3\sqrt{16}\sqrt{5} = 2\sqrt{5} + 6\sqrt{5} - 12\sqrt{5} = -4\sqrt{5}$$

$$5. a) \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$b) \frac{5}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{10+5\sqrt{3}}{4-2\sqrt{3}+2\sqrt{3}-\sqrt{3}\sqrt{3}} = \frac{10+5\sqrt{3}}{4-3} = 10 + 5\sqrt{3}$$

$$6. a) (x-8)(x+3), \text{ so } x = 8, x = -3$$

$$b) (3x+2)(3x-2) \text{ Difference of two squares, So } x = -\frac{2}{3} \text{ or } x = \frac{2}{3}$$

$$7. a) x = \frac{-1 \pm \sqrt{1-4(6)(-1)}}{2(6)} = \frac{-1 \pm \sqrt{1+24}}{12} = \frac{-1 \pm \sqrt{25}}{12} = \frac{-1 \pm 5}{12}$$

$$x = \frac{-1+5}{12} = \frac{1}{3} \text{ or } x = \frac{-1-5}{12} = -\frac{1}{2}$$

$$b) x = \frac{7 \pm \sqrt{49-4(1)(9)}}{2} = \frac{7 \pm \sqrt{49-36}}{2} = \frac{7 \pm \sqrt{13}}{2}, \quad x = \frac{7+\sqrt{13}}{2} \text{ or } x = \frac{7-\sqrt{13}}{2}$$

$$8. a) (x+1)^2 - 1 - 6 = (x+1)^2 - 7$$

$$b) \left(x + \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 + \frac{1}{4} = \left(x + \frac{3}{2}\right)^2 - \frac{9}{4} + \frac{1}{4} = \left(x + \frac{3}{2}\right)^2 - 2$$

$$9. a) \cos 70^\circ = \frac{x}{24}, \quad x = 24 \cos 70^\circ = 8.2$$

$$b) \text{ Cosine Rule: } a^2 = 3^2 + 15^2 - 2(3)(15) \cos 150^\circ = 311.9, \quad a = 17.7$$

$$10. a) 3^3 \times 3^n = 3^{3+n} \text{ (Index Rule)}$$

$$b) 27 \times (3^{n+1}) = 3^3 \times (3^{n+1}) = 3^{n+4} \text{ (Index Rules)}$$

Staple your completed test to this booklet and bring it with you to your first lecture in September so you have a record that you can discuss with your lecturer.

Any issues, please use the video solutions and links provided to revise and relearn topics so that you are fully prepared to start A Level Maths in September.